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	supported by the Office of Naval Research, the National Science Foundation, the Army Research Office, the Air Force Office of Scientific Research, and Stanford University. The personnel comprising this group during all or part of the reporting period are Joseph B. Keller, Professor of Mathematics, Stanford University; Russel E. Caflisch, Assistant							
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ITEM #11, TITLE: PROGRESS REPORT, GRANT AFOSR-79-0134, SEPTEMBER 1, 1982 - APRIL 30, 1983.

ITEM #19, ABSTRACT, CONTINUED: University; Margaret Cheney, Research Associate, Stanford University; Graham Eatwell, Postdoctoral Fellow, Stanford University; Luis Lopez-Bonilla, Research Associate, Stanford University; James Geer, Professor of Mathematics, School of Advanced Studies, SUNY, Center at Binghamton, New York; Michael Reeken, Professor of Mathematics, University of Wuppertal, Germany; Erik Hansen, Professor of Mathematics, Technical University of Denmark, Lyngby. The various research activities of the members of this group are indicated by the list of publications contained in Section II. or introductions from the papers produced during the report period are contained in Section III. In addition to the manuscripts already submitted, a number of projects are nearing Thus Professor Keller, while at the Woods Hole Oceanographic Institute during the summer of 1982, studied the turbulent diffusion of a chemically reacting substance. Professor Keller and Dr. Falkovitz are completing a paper on Liesegang rings to account for the recent results of John Ross and his co-worker in the Chemistry Department at Stanford. Their theory uses the fourth order Cahn-Hilliard equation to describe diffusion. Dr. Eatwell is studying the propagation of an acoustic wave through a bubbly fluid, or through a medium containing resonant scatterers. Dr. Bonilla is studying the effective behavior of an elastic solid composed of randomly oriented cry-tallites. Mr. Nunan is calculating the effective viscosity of a suspension of rigid spherical particles in a viscous fluid and the effective elasticity of a lattice of rigid spheres in an elastic solid. Professors Geer and Keller are studying scattering of waves by thin bodies. In December 1982 Professor Keller was invited to deliver the Weizmann Lectures at the Weizmann Institute of Science in Rohovot, Israel. In March 1983 he presented an invited lecture at the British Theoretical Mechanics Colliquium in Manchester, England. In April 1983 he spent a week as Distinguished Visiting Professor of Mechanics and Applied Mathematics at Cornell University.

PROGRESS REPORT, Grant AFOSR-79-0134

September 1, 1982 - April 30, 1983

Applied Mathematics Group Department of Mathematics Stanford University

Principal Investigator: Joseph B. Keller

## 1. Introduction

This is a progress report of the Applied Mathematics Group in the Mathematics Department, Stanford University. This group began functioning officially on September 1, 1979, and is supported by:

- Office of Naval Research; 1.
- National Science Foundation:
- Army Research Office;
- Air Force Office of Scientific Research;
- 5. Stanford University.

The personnel comprising this group during all or part of the reporting period are:

- Joseph B. Keller, Professor of Mathematics, Stanford University;
- Russel E. Caflisch, Assistant Professor, Stanford University; 2.
- John H. Maddocks, Acting Assistant Professor, Stanford University; 3.
- Meira S. Falkovitz, Postdoctoral Fellow, Stanford University;
- Victor Twersky, Professor of Mathematics, University of Illinois, Chicago Circle;
  - 6. John G. Watson, Assistant Professor, University of Miami;

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- 7. Bernard A. Lippmann, Professor of Physics, Emeritus, New York University;
  - 8. Kevin C. Nunan, Graduate Student;
  - 9. Michael I. Weinstein, Postdoctoral Fellow, Stanford University;
  - 10. Stephanos Venakides, Assistant Professor, Stanford University;
  - 11. Margaret Cheney, Research Associate, Stanford University;
  - 12. Graham Eatwell, Postdoctoral Fellow, Stanford University;
  - 13. Luis Lopez-Bonilla, Research Associate, Stanford University;
- 14. James Geer, Professor of Mathematics, School of Advanced Studies, SUNY, Center at Binghamton, New York.
- 15. Michael Recken, Professor of Mathematics, University of Wuppertal, Germany.
- 16. Erik Hansen, Professor of Mathematics, Technical University of Denmark, Lyngby.

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Joseph B. Keller Progress Report Page 3

recent results of John Ross and his co-worker in the Chemistry Department at Stanford. Their theory uses the fourth order Cahn-Hilliard equation to describe diffusion. Dr. Eatwell is studying the propagation of an acoustic wave through a bubbly fluid, or through a medium containing resonant scatterers. Dr. Bonilla is studying the effective behavior of an elastic solid composed of randomly oriented crystallites. Mr. Numan is calculating the effective viscosity of a suspension of rigid spherical particles in a viscous fluid and the effective elasticity of a lattice of rigid spheres in an elastic solid. Professors Geer and Keller are studying scattering of waves by thin bodies.

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## 11. RECENT PUBLICATIONS OF THE APPLIED MAINEMATICS GROUP

50.	J. B. Keller	Opt rum inspection policies
		Pub: Management Sci., 28, 447-450, 1982.
51.	P. S. Hagan	Travelling and stacked travelling wave solutions of parabolic equations
		Pub: SIAM J. Math. Anal., 13, 717-738, 1982.
52.	JM. Vanden-Broeck	Contact problems involving the flow past an inflated aerofoil
		<u>Pub</u> : J. Appl. Mech., <u>49</u> , 263-265, 1982.
53.	JM. Vanden-Broeck	Nonlinar two-dimensional sail theory
		<u>Pub</u> : Phys. Fluids, <u>25</u> , 420-423, 1982.
55.	P. S. Hagan	Spiral waves in reaction diffusion equations
		Pub: SIAM J. Appl. Math., 42, 762-786, 1982.
56.	JM. Vanden-Broeck J. B. Keller	Parabolic approximations for ship waves and wave resistance
		Pub: Proceedings of the Third International Conference on Numerical Ship Hydrodynamics, Paris, France, June 16-19, 1981.
57.	A. Jeffrey	Asymptotic Methods in Nonlinear Wave Problems
	T. Kawahara	Pub: Pitman Publishing, Ltd., London, 1982.
58.	M. J. Miksis	Rising Bubbles
	JM. Vanden-Broeck J. B. Keller	Pub: J. Fluid Mech., 123, 31-42, 1982.
59.	J. C. Neu	Resonantly interacting waves
		Tab: SIAM J. Appl. Math., 43, 1, 141-156, 1983.
60.	J. B. Keller	Surface tension driven flows
	M. J. Miksis	Pub: SIAM J. Appl. Math., 43, 11, 268-277, 1983.
61.	A. Jepson A. Spence	Folds in solutions of two parameter systems and their calculation: Part I
		Pub: Stanford Univ. Numer. Anal. Report,

82-02.

62.	J. B. Keller	Tim dependent queues  Pub: SIAM Rev., 24, 401-412, 1982.
63.	R. E. Caflisch B. Nicolaenko	Shock profile solutions of the Boltzmann equation  Pub: Comm. Math. Phys., 86, 161-194, 1982.
64.	P. S. Hagan R. E. Caflisch J. B. Keller	Arrow's model of optimal pricing, use and exploration of undertain natural resources  Sub: Econometrica
65.	R. E. Caflisch	Radiation transport in a hot plasma Acc: SIAM J. Appl. Math., in press.
66.	J. B. Keller JM. Vanden-Broeck	Jets rising and falling under gravity  Pub: J. Fluid Mech., 124, 335-345, 1982.
67.	R. E. Caflisch	Fluid dynamics and the Boltzmann equation <u>Pub</u> : Stud. Stat. Mech., <u>5</u> , 194-223, 1983.
68 <b>.</b> *	M. S. Falkovitz M Soul	Theory of periodic structures in lipid bilayer membranes
	H. L. Frisch H. M. McConnell	<u>Pub</u> : Proc. Nat. Acad. Sci., <u>79</u> , 3918, 1982.
69.	R. E. Caflisch	The fluid-dynamic limit of a model Boltzmann equation in the presence of a shock
		Pub: Institute National de Recherche en Informatique et en Automatique, INRIA No. 81, June 1981, 1-34.
70.	P. F. Rhodes-Robinson	On the short surface waves due to half-immersed circular cylinder oscillating on water of infinite depth
		Pub: Proc. Royal Soc. London Λ, 384, 333-357, 1982.
71.	P. F. Rhodes-Robinson	Note on the reflexion of water waves at a wall in the presence of surface tension
		Pub: Math. Proc. Cambridge Philosophical Soc., 92, 369-373, 1982.

<sup>\*</sup>Not supported by AFOSR or ONR.

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72.	P. F. Rhodes-Robinson	On the generation of water waves at an inertial surface		
		Sub.		
73.	R. E. Caflisch G. C. Papanicolaou	Dynamic theory of suspensions with Brownian effects		
		Ace: SIAM J. Appl. Math., in press.		
74.	R. E. Caflisch G. C. Papanicolaou	Instability in settling of suspensions due to Brownian effects		
		Pub: Proceedings of conference Two-Phase Flow.		
75.	R. E. Caflisch	Shock waves and the Boltzmann equation		
	B. Nicolaenko	Acc: Proceedings NSF-AMS conference non-linear PDE.		
76.	J. H. Maddocks	Restricted quadratic forms and their application to bifurcation and stability in constrained variational principles		
		Sub: SIAM J. Appl. Math.		
77.	M. S. Falkovitz L. A. Segel	Spatially inhomogeneous polymerization in unstirred bulk		
		<u>Pub</u> : SIAM J. Appl. Math., <u>43</u> , 386-416, 1983.		
78.	M. S. Falkovitz J. L. Frisch	The scale of non-homogeneity as defined by diffusion measurements		
		Pub: Journal of Membrane Science, 10, 61, 1982.		
79.	M. S. Falkovitz	Optimal catalyst distribution in a membrane		
	H. L. Frisch J. B. Keller	Acc: Chem. Eng. Sci.		
80.3	8 M. S. Falkovitz	Crawling of Worms		
	J. B. Keller	Acc: J. Theor. Biol.		
81.	A. Jeffrey	The random choice method and the free-surface		

water profile after the collapse of a dam wall

Pub: Wave Motion, 4, 381-389, 1982.

J. Myungi

<sup>\*</sup>Not supported by AFOSR or ONR.

82.	J. G. Watson	As stistical theory for imperfect bifurcation
	E. L. Reiss	Pub SIAM J. Appl. Math., 42, 135-148, 1982.
83.	J. G. Watson J. B. Keller	Reffection, scattering and absorption of acoustic waves by rough surfaces
		To be submitted.
84.	M. l. Weinstein	Global existence for a generalized Korteweg - de Vries equation
		Sub: SIAM J. Math. Anal.
85.	M. I. Weinstein	Nonlinear Schrödinger equations and sharp interpolation estimates
		Pub: Commun. Math. Phys., 87, 567-576, 1983.
97	M. Change	The dimensional mathematics the number of
86.	M. Cheney	Two-dimensional scattering: the number of bound states from scattering data
		Sub: J. of Math. Phys.
87.	L. L. Bonilla A. Liñán	Relaxation oscillations, pulses, and travelling waves in the diffusive Volterra delay-differential equation
		Acc: SIAM J. of Appl. Math., in press.
88.	P. F. Rhodes-Robinson	Note on the effect of surface tension on water waves at an inertial surface
		<u>Pub</u> : J. Fluid Mech., <u>125</u> , 375-377, 1982.
89.	J. B. Keller	Weak shock diffraction
0,1	J. Hunter	Acc: Wave Motion, in press.
90.	J. B. Keller	Weakly nonlinear high frequency waves
	J. Hunter	Acc: Comm. Pure and Appl. Math., in press.
91.	J. B. Ecller	Asymptotic analysis of a viscous Cochlear model
	J. C. Neu	Sub: J.A.S.A.
92.	J. B. Keller JH. Vanden-Broeck	Parabolic approximations for ship waves and wave resistance

Pub:

Proc. 3rd Intl. Conf. on Numerical Ship

Hydrodynamics, Paris, France, June 16-19, 1981.

93.	A. Spence A. Jepson	The nurrical computation of turning points of nonline (requations  Pub: Treatment of Integral Equations by Numerical
		Methods, 169-183, London, 1982.
94.	J. B. Keller R. Burridge	Biot's poroelasticity equations by homogenization  Pub: Springer Lecture Notes, in Macroscopic Properties of Disordered Media, NY, 51-57,1982.
95.	J. B. Keller	Capillary waves on a vertical jet  Acc: J. Fluid Mech., in press.
		, , , , , , , , , , , , , , , , , , , ,
96.	J. B. Keller	Survival estimation using censored data
	A. S. Whittemore	Sub: J. Royal Statist. Soc., Series B
97.	J. B. Keller J. F. Geer	Eigenvalues of slender cavities and waves in slender tubes
		To be submitted.
98.	J. B. Keller	Valuation of stocks and options
20.	R. Voronka	·
		To be submitted.

1. Dynamic theory of suspensions with Brownian effects, by R. E. Caflisch and G. C. Papanicolaou.

We consider a suspension of particles in a fluid settling under the influence of gravity and dispersing by Brownian motion. A mathematical description is provided by the Stokes equations and a Fokker-Planck equation for the one-particle phase space density. This is a nonlinear system that depends on a number of parametric functions of the spatial concentration of the particles. These functions are known only empirically or for additions. We analyze the system, its stability, its asymptotic behavior under different scalings and its validity from a more microscopic description.

2. Two-dimensional scattering: the number of bound states from scattering data, (a), by M. Cheney.

Relations are found between scattering data and the spectrum for the two-dimensional Schrödinger operator  $\Delta+V(x)$ , where V is a local noncentral potential. In particular a two-dimensional version of the Levinson theorem is obtained; this theorem gives the number of bound states in terms of the change in phase of the determinant of the scattering operator.

3. Shock profile solutions of the F 'thmann equation, by R. E. Crilis a and B. Nicolaenko.

Shock waves in gas dynamics can be described by the Euler Marier-Stages, or Boltzmann equations. We prove the existence of shock profile solutions at the Boltzmann equation for shocks which are weak. The shock is written as a truncated expansion in powers of the shock strength, the first two terms of which cone exactly from the Taylor tanh (x) profile for the Waviscott solution. The full solution is found by a projection method like the hapanex-Schmidt method as a bifurcation from the constant state in which the Promoter series and parameter is the difference between the speed of sound  $\varepsilon_{ij}$  and the same as speed so.

4. Restricted quadratic forms and their application to bifurcation as a dility in constrained variational principles, by J. H. Maddecks.

The subjects of this investigation are the abstract properties and upplications of restricted pullettic forms. The first part of the presentation resolves the following question: If L is a self-adjoint linear operator mapping a Hilbert space H into itself, and S is a subspace of H, when is the quadratic form  $\leq u_1$ Lub positive for any non-zero  $u \in S$ ? In the second part of the presentation, restricted quadratic forms are further examined in the specific context of constrained variational principles; and the general theory is applied to obtain information on stability and bifurcation. Two examples are then solved: one is finite-dimensional and of an illustrative nature; the other is a longstanding problem in elasticity concerning the stability of a buckled rod. In addition to being a valuable analytical tool for isoperizetric problems in the calculus of variations, the tests described are assemble to magnetical treatment. A theorem concerning the variational characterization of eigenvalues is also obtained.

5. Spatially inhomogeneous polymerization in unstirred back, by H. S. (  $\mbox{\it Like}$  and L. A. Segel.

A new experimental method for the formation of large polymers of very

belongs to the general class of transport-reaction problems in multi-component media. A comprehensive mathematical model of the phenomenon is presented. Noteworthy is the fact that diffusive fluxes are decomposed into two parts, pure diffusion and back-bulk-flow. It is shown that the full problem, which consists of an infinite number of coupled nonlinear differential equations, can be reduced to two such equations. Heat production is also considered in an auxiliary investigation. The appropriate differential equations are solved numerically and the physical significance of the results is discussed.

6. The scale of non-homogeneity as defined by diffusion measurements, by M. S. Falkovitz and H. L. Frisch.

 $1/\ell^2$ . The result is compared with previous studies of this subject.

The asymptotic behaviour of the permeation rate in non-homogeneous laminar slabs of increasing width t is studied. We show rigorously that for a wide class of slabs with non-homogeneities distributed in a statistically homogeneous manner, the permeation rate approaches that of a homogeneous slab as

7. Crawling of worms, by J. B. Keller and M. S. Falkovitz.

The mechanics of a worm crawling along a flat surface is analyzed. The external forces of friction and gravity, and the internal pressure and tension, are taken into account. An equation of motion is formulated, and solutions are sought in which both the tension and the linear density are required to lie between prescribed bounds. Pulse and periodic travelling wave solutions are constructed. The maximum crawling velocity is determined, as well as the wave form which achieves it. Comparison of the results with experimental observations shows that the theory yields a maximum crawling velocity which is much larger than the observed velocity. Therefore the theory was changed to require that the time rate of change of tension be less than a prescribed bound, rather than

that the traction be bounded. With this modification, the theory agrees fairly well with the observations.

8. The random choice method and the free-surface water profile after the collapse of a dam wall, by  $\Lambda$ . Jeffrey and J. Hvungi.

The random choice (RC) method due to Glimm [1], subsequently modified by Chorin [2] and Sod [3], is applied to the shallow water equations of Stoker [4], in order to determine the free-surface profile of water, as a function of position and time, when a dam wall soldenly collapses. The numerical results obtained by this method are compared with the analytical results due to Stoker for this classical problem [5], and with numerical results obtained by a further modification of the Glimm-Chorin scheme involving a smoothing operation. The improvement brought about by the smoothing, and the generally close agreement with the exact result that is then obtained, is a feature of this modified approach.

9. A statistical theory for imperfect bifurcation, by J. G. Watson and E. L. Reiss.

An "honest" statistical method is presented to analyze the effects of imperfections, and other disturbances on the bifurcation of solutions of non-linear problems. First, uniformly valid asymptotic approximations of the solutions are obtained for any realization of the imperfections. The approximations are valid as the magnitude of the imperfections approaches zero. The statistical properties of the solutions are then deduced directly from there approximations, for specified statistics of the imperfections. For simplicity of presentation, the imperfections are taken as zero-mean, wide-sense stationary, Gaussian random processes. The statistical analysis is elementary. It provides easily analyzed results for the expected values and variances of the solutions. Confidence limits are also given.

10. Global existence for a generalized Korteweg - de Vries equation, by M. Weinstein.

A sufficient condition for global existence is obtained for the generalized Korteweg — de Vries Equation (GKdV)  $u_1 + u_2^{\dagger}u_2 + u_{_{\rm KKK}} = 0$ ,  $\kappa_0 R^{\dagger}$ ,  $\kappa_0 R^{\dagger}$ . This condition is expressed in terms of the solitary (traveling) wave solution of GKdV.

11. Nonlinear Schrödinger equations and sharp interpolation estimates, by M. I. Weinstein.

A sharp sufficient condition for global existence is obtained for the nonlinear Schrödinger equation

(NLS) 
$$2i\psi_{t} + \Delta\phi + |\phi|^{2\sigma}\phi = 0$$
,  $x \in \mathbb{R}^{N}$ ,  $t \in \mathbb{R}^{+}$ ,

in the case  $\sigma=2/N$ . This condition is in terms of an exact stationary solution (nonlinear ground state) of (NLS). It is derived by solving a variational problem to obtain the "best constant" for classical interpolation estimates of Nirenberg and Gagliardo.

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